Reasoning on issues: one logic and a half

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Overview

1. Dichotomous inquisitive logic:
   reasoning with issues

2. Inquisitive epistemic logic:
   reasoning about entertaining issues

3. Inquisitive dynamic epistemic logic:
   reasoning about raising issues

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3. ~~Inquisitive dynamic epistemic logic:~~
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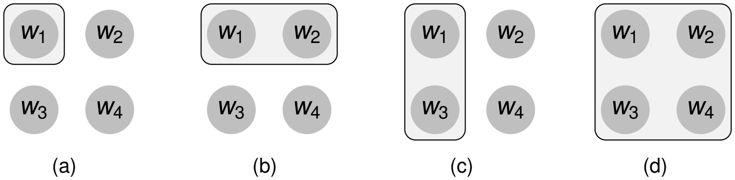
Preliminaries

Information states

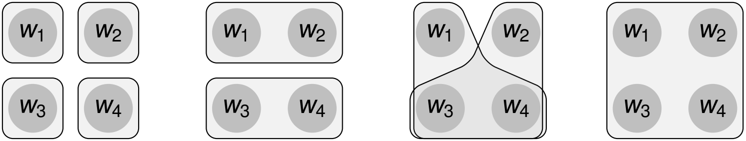
- Let $\mathcal{W}$ be a set of possible worlds.
- Definition: an information state is a set of possible worlds.
- We identify a body of information with the worlds compatible with it.
- t is at least as informed as s in case $t \subseteq s$.
- The state $\emptyset$ compatible with no worlds is called the absurd state.



Preliminaries

Issues

- Definition: an issue is a non-empty, downward closed set of states.
- An issue is identified with the information needed to resolve it.
- An issue $\mathcal{I}$ is an issue over a state s in case $s = \bigcup \mathcal{I}$.
- The alternatives for an issue $\mathcal{I}$ are the maximal elements of $\mathcal{I}$.



(e) (f) (g) (h)

Four issues over $\{w1, w2, w3, w4\}$: only alternatives are displayed.

Part I

Dichotomous inquisitive logic: reasoning with issues

Dichotomous inquisitive semantics

Definition (Syntax of InqD_π)

$\mathcal{L}_{InqD_\pi}$ consists of a set $\mathcal{L}_!$ of declaratives and a set $\mathcal{L}_?$ of interrogatives:

1. if $p \in \mathcal{P}$, then $p \in \mathcal{L}_!$
2. $\bot \in \mathcal{L}_!$
3. if $\alpha_1, \ldots, \alpha_n \in \mathcal{L}_!$, then $?\{\alpha_1, \ldots, \alpha_n\} \in \mathcal{L}_?$
4. if $\varphi, \psi \in \mathcal{L}_\circ$, then $\varphi \wedge \psi \in \mathcal{L}_\circ$
5. if $\varphi \in \mathcal{L}_! \cup \mathcal{L}_?$ and $\psi \in \mathcal{L}_\circ$, then $\varphi \to \psi \in \mathcal{L}_\circ$

Abbreviations

- if $\alpha \in \mathcal{L}_!$, $\neg\alpha := \alpha \to \bot$
- if $\alpha, \beta \in \mathcal{L}_!$, $\alpha \vee \beta := \neg(\neg\alpha \wedge \neg\beta)$
- if $\alpha \in \mathcal{L}_!$, $?\alpha := ?\{\alpha, \neg\alpha\}$

Dichotomous inquisitive semantics

Notational convention on meta-variables

	Declaratives	Interrogatives	Full language
Formulas	α, β, γ	μ, ν, λ	φ, ψ, χ
Sets of formulas	Γ	Λ	Φ

Dichotomous inquisitive semantics

Semantics

- Usually, the role of semantics is to assign truth-conditions.
- However, our language now contains interrogatives as well.
- Claim: interrogative meaning = resolution conditions.
- We could give a double-face semantics: truth-conditions at worlds for declaratives, resolution conditions at info states for interrogatives.
- Instead, we will lift everything to the level of information states.
- Our semantics is defined by a relation $\models$ of support between information states and formulas, where:

  Declaratives: $s \models \alpha \iff \alpha$ is established in s

  Interrogatives: $s \models \mu \iff \mu$ is resolved in s

Dichotomous inquisitive semantics

Definition (Models)

A model for a set $\mathcal{P}$ of atoms is a pair $M = \langle \mathcal{W}, V \rangle$ where:

- $\mathcal{W}$ is a set whose elements are called possible worlds
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a valuation function

Definition (Support)

Let M be a model and let s be an information state.

1. $M, s \models p \iff p \in V(w)$ for all worlds $w \in s$
2. $M, s \models \bot \iff s = \emptyset$
3. $M, s \models ?\{\alpha_1, \dots, \alpha_n\} \iff M, s \models \alpha_1$ or $\dots$ or $M, s \models \alpha_n$
4. $M, s \models \varphi \wedge \psi \iff M, s \models \varphi$ and $M, s \models \psi$
5. $M, s \models \varphi \to \psi \iff$ for any $t \subseteq s$, if $M, t \models \varphi$ then $M, t \models \psi$

Dichotomous inquisitive semantics

Fact (Perstistence)

If $M, s \models \varphi$ and $t \subseteq s$ then $M, t \models \varphi$.

Fact (Absurd state)

$M, \emptyset \models \varphi$ for any formula φ and model M.

Definition (Proposition)

The proposition expressed by φ in M is the set of states supporting φ:

$$[\varphi]_M = \{s \subseteq \mathcal{W} \mid s \models \varphi\}$$

Fact (Propositions are issues)

$[\varphi]_M$ is an issue for any formula φ and model M.

Dichotomous inquisitive semantics

Definition (Truth)

$M, w \models \varphi \overset{def}{\iff} M, \{w\} \models \varphi$

Definition (Truth-set)

$|\varphi|_M := \{w \in \mathcal{W} \mid M, w \models \varphi\}$

Fact (Truth and support)

$|\varphi|_M = \bigcup [\varphi]_M$

Dichotomous inquisitive semantics

Fact (Truth-conditions)

- $M, w \models p \iff p \in V(w)$
- $M, w \not\models \bot$
- $M, w \models ?\{\alpha_1, \ldots, \alpha_n\} \iff M, w \models \alpha_1$ or $\ldots$ or $M, w \models \alpha_n$
- $M, w \models \varphi \wedge \psi \iff M, w \models \varphi$ and $M, w \models \psi$
- $M, w \models \varphi \rightarrow \psi \iff M, w \not\models \varphi$ or $M, w \models \psi$

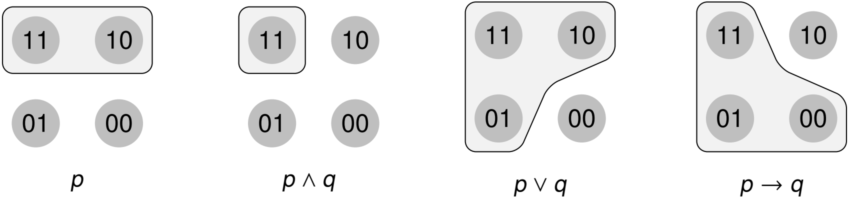
Dichotomous inquisitive semantics

Truth for declaratives

- The semantics of a declarative is determined by truth conditions:

$$M, s \models \alpha \iff \text{for all } w \in s,\ M, w \models \alpha$$

- That is, we always have $[\alpha]_M = \wp(|\alpha|_M)$
- Since truth-conditions are standard, declaratives are classical.



Dichotomous inquisitive semantics

Truth for interrogatives

$$M, w \models \mu \iff w \in s \text{ for some } s \models \mu$$
$$\iff w \in s \text{ for some } s \text{ resolving } \mu$$
$$\iff \mu \text{ can be truthfully resolved in } w$$

Definition (Presupposition of an interrogative)

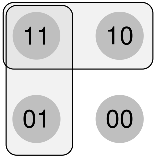
- $\pi_{?\{\alpha_1,\ldots,\alpha_n\}} = \alpha_1 \vee \cdots \vee \alpha_n$
- $\pi_{\mu \wedge \nu} = \pi_\mu \wedge \pi_\nu$
- $\pi_{\varphi \to \mu} = \varphi \to \pi_\nu$

Fact

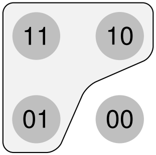
$|\mu|_M = |\pi_\mu|_M$

Remark

For interrogatives, truth-conditions do not fully determine meaning. Ex. consider $?p$ and $?q$.



$[?\{p, q\}]$

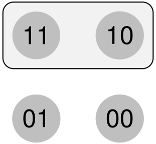


$|?\{p, q\}|$

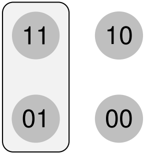
Dichotomous inquisitive semantics

Conjunction

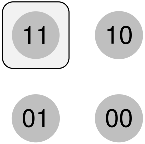
$M, s \models \varphi \wedge \psi \iff M, s \models \varphi$ and $M, s \models \psi$



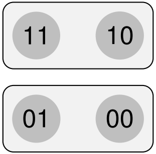
p



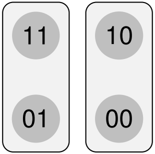
q



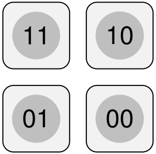
$p \wedge q$



$?p$



$?q$

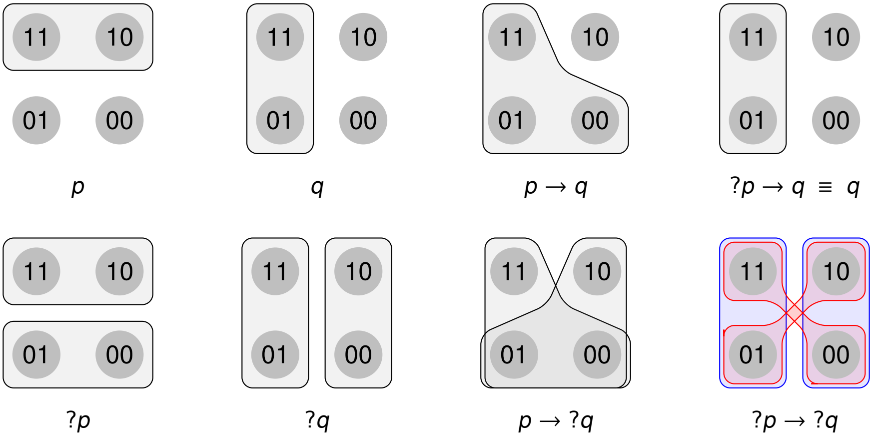


$?p \wedge ?q$

Dichotomous inquisitive semantics

Implication

$$M, s \models \varphi \to \psi \iff \text{for any } t \subseteq s, \text{ if } M, t \models \varphi \text{ then } M, t \models \psi$$



Dichotomous inquisitive logic

Definition (Entailment)

$\Phi \models \psi \iff$ for all M, s, if $M, s \models \Phi$ then $M, s \models \psi$

Declarative conclusion

$\Gamma, \Lambda \models \alpha \iff$ establishing Γ and Π_Λ implies establishing α.

Interrogative conclusion

$\Gamma, \Lambda \models \mu \iff$ establishing Γ and resolving Λ implies resolving μ.

Dichotomous inquisitive logic

Example 1

- $p \leftrightarrow q \wedge r, \quad ?q \wedge ?r \quad \models \quad ?p$
- $p \leftrightarrow q \wedge r, \quad ?p \quad \not\models \quad ?q \wedge ?r$

Example 2

- $?p \rightarrow ?q, \quad ?p \quad \models \quad ?q$

Dichotomous inquisitive logic

Four particular cases

- $\alpha \models \beta \iff \alpha$ is at least informative as β
- $\alpha \models \mu \iff \alpha$ resolves μ
- $\mu \models \alpha \iff \mu$ presupposes α
- $\mu \models \nu \iff \mu$ is at least as inquisitive as ν

Dichotomous inquisitive logic

Conjunction

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta} \qquad \frac{\alpha \wedge \beta}{\alpha} \qquad \frac{\alpha \wedge \beta}{\beta}$$

Implication

$$\frac{\begin{matrix}[\alpha] \\ \vdots \\ \beta\end{matrix}}{\alpha \to \beta} \qquad \frac{\alpha \quad \alpha \to \beta}{\beta}$$

Falsum

$$- \qquad \frac{\bot}{\alpha}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Dichotomous inquisitive logic

Conjunction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

Implication

$$\frac{\begin{matrix}[\varphi] \\ \vdots \\ \psi\end{matrix}}{\varphi \to \psi} \qquad \frac{\varphi \quad \varphi \to \psi}{\psi}$$

Falsum

$$- \qquad \frac{\bot}{\varphi}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Dichotomous inquisitive logic

Conjunction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

Implication

$$\frac{\begin{matrix}[\varphi] \\ \vdots \\ \psi\end{matrix}}{\varphi \to \psi} \qquad \frac{\varphi \quad \varphi \to \psi}{\psi}$$

Falsum

$$\frac{\bot}{\varphi}$$

Interrogative

$$\frac{\alpha_i}{?\{\alpha_1, \dots, \alpha_n\}} \qquad \frac{\begin{matrix}[\alpha_1] \\ \vdots \\ \varphi\end{matrix} \quad \dots \quad \begin{matrix}[\alpha_n] \\ \vdots \\ \varphi\end{matrix} \quad ?\{\alpha_1, \dots, \alpha_n\}}{\varphi}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Dichotomous inquisitive logic

Conjunction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

Implication

$$\frac{\begin{matrix}[\varphi] \\ \vdots \\ \psi\end{matrix}}{\varphi \to \psi} \qquad \frac{\varphi \quad \varphi \to \psi}{\psi}$$

Falsum

$$- \qquad \frac{\bot}{\varphi}$$

Interrogative

$$\frac{\alpha_i}{?\{\alpha_1, \ldots, \alpha_n\}} \qquad \frac{\begin{matrix}[\alpha_1] \\ \vdots \\ \varphi\end{matrix} \quad \ldots \quad \begin{matrix}[\alpha_n] \\ \vdots \\ \varphi\end{matrix} \quad ?\{\alpha_1, \ldots, \alpha_n\}}{\varphi}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Kreisel-Putnam axiom

$$(\alpha \to ?\{\beta_1, \ldots, \beta_m\}) \;\to\; ?\{\alpha \to \beta_1, \ldots, \alpha \to \beta_m\}$$

Dichotomous inquisitive logic

Definition (Resolutions)

To any formula φ we associate a set of declaratives called resolutions.

- $\mathcal{R}(\alpha) = \{\alpha\}$ if α is a declarative
- $\mathcal{R}(?\{\alpha_1, \dots, \alpha_n\}) = \{\alpha_1, \dots, \alpha_n\}$
- $\mathcal{R}(\mu \wedge \nu) = \{\alpha \wedge \beta \,|\, \alpha \in \mathcal{R}(\mu)$ and $\beta \in \mathcal{R}(\nu)\}$
- $\mathcal{R}(\varphi \to \mu) = \{\bigwedge_{\alpha \in \mathcal{R}(\varphi)} \alpha \to f(\alpha) \,|\, f : \mathcal{R}(\varphi) \to \mathcal{R}(\mu)\}$

Resolutions of a set

Replace each element in the set by one or more resolutions:

$$\mathcal{R}(\ \{p,\ \ ?q \wedge ?r\}\ ) \quad = \quad \{ \quad \begin{array}{l} \{p,\ \ q \wedge r\} \\ \{p,\ \ q \wedge \neg r\} \\ \dots \\ \dots \end{array} \quad \}$$

Dichotomous inquisitive logic

Theorem (Resolution theorem)

$$\Phi \vdash \psi \quad \Longleftrightarrow \quad \forall \Gamma \in \mathcal{R}(\Phi) \quad \exists \alpha \in \mathcal{R}(\psi) \text{ s.t. } \Gamma \vdash \alpha$$

Corollary

There exists an effective procedure that,
when given as input:

- a proof of $\Phi \vdash \psi$
- a resolution Γ of Φ

outputs:

- a resolution α of ψ
- a proof of $\Gamma \vdash \alpha$

Dichotomous inquisitive logic

Example

If we feed the algorithm

- a proof of $p \leftrightarrow q \wedge r,\ \ ?q \wedge ?r \ \vdash\ ?p$
- the resolution $p \leftrightarrow q \wedge r,\ \ q \wedge \neg r$

It will return

- the resolution $\neg p$ of $?p$
- a proof of $p \leftrightarrow q \wedge r,\ \ q \wedge \neg r \ \vdash\ \neg p$

Dichotomous inquisitive logic

Definition (Canonical model)

The canonical model for InqD_π is the model $M^c = \langle \mathcal{W}^c, V^c \rangle$ where:

- $\mathcal{W}^c$ consists of complete theories of declaratives
- $V^c : \mathcal{W}^c \to \wp(\mathcal{P})$ is defined by $V^c(\Gamma) = \{p \mid p \in \Gamma\}$

Lemma (Support lemma)

For any $S \subseteq \mathcal{W}^c$, $\quad M^c, S \models \varphi \iff \bigcap S \vdash \varphi$

Theorem (Completeness)

$\Phi \models \psi \iff \Phi \vdash \psi$

Part II

Reasoning about entertaining issues: Inquisitive Epistemic Logic

Inquisitive epistemic logic

Epistemic Logic

In standard EL we can reason about facts and (higher-order) information.

Inquisitive Epistemic Logic

In IEL we can reason about facts, information and issues, including the higher-order cases:

- information about information
- information about issues
- issues about information
- issues about issues

Inquisitive epistemic logic

Standard epistemic models

An epistemic model is a triple $M = \langle \mathcal{W}, V, \{\sigma_a(w) \mid a \in \mathcal{A}\} \rangle$ where:

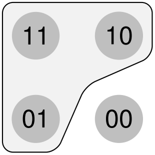
- $\mathcal{W}$ is a set of possible worlds
- $V : \mathcal{W} \rightarrow \wp(\mathcal{P})$ is a valuation function
- $\sigma_a : \mathcal{W} \rightarrow \wp(\mathcal{W})$ is the epistemic map of agent a, delivering for any w an information state $\sigma_a(w)$, in accordance with:

  Factivity : $w \in \sigma_a(w)$

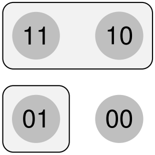
  Introspection : if $v \in \sigma_a(w)$ then $\sigma_a(v) = \sigma_a(w)$

Inquisitive epistemic logic

- We want to add a description of the issues agents entertain.
- Replace the epistemic maps σ_a by a state map Σ_a that describes both information and issues.
- For any world w, $\Sigma_a(w)$ delivers an issue:
  - the information of the agent is $\sigma_a(w) = \bigcup \Sigma_a(w)$
  - the agent wants to reach one of the states $t \in \Sigma_a(w)$



$\sigma_a(w)$



$\Sigma_a(w)$

Inquisitive epistemic models

Definition (Inquisitive epistemic models)

An inquisitive epistemic model is a triple $\langle \mathcal{W}, V, \{\Sigma_a \mid a \in \mathcal{A}\}\rangle$, where:

- $\mathcal{W}$ is a set of possible worlds
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a valuation function
- Σ_a is the state map of agent a, delivering for any w an issue $\Sigma_a(w)$, in accordance with:

  Factivity : $w \in \sigma_a(w)$

  Introspection : if $v \in \sigma_a(w)$ then $\Sigma_a(v) = \Sigma_a(w)$

  where $\sigma_a(w) := \bigcup \Sigma_a(w)$.

Inquisitive epistemic logic

Definition (Syntax)

The language $\mathcal{L}_{\mathsf{IEL}}$ for a set $\mathcal{A}$ of agents is obtained expanding $\mathcal{L}_{InqD_\pi}$ with the following clauses:

- if $\varphi \in \mathcal{L}_! \cup \mathcal{L}_?$ and $a \in \mathcal{A}$, then $K_a\varphi \in \mathcal{L}_!$
- if $\varphi \in \mathcal{L}_! \cup \mathcal{L}_?$ and $a \in \mathcal{A}$, then $E_a\varphi \in \mathcal{L}_!$

Remark

Notice that now the definitions of $\mathcal{L}_!$ and $\mathcal{L}_?$ are intertwined:

- the interrogative operator ? forms interrogatives out of declaratives;
- the modalities K_a and E_a form declaratives out of interrogatives;
- we can thus form sentences such as $E_a?K_b?p$.

Inquisitive epistemic logic

Definition (Support conditions for the modalities)

- $M, s \models K_a\varphi \iff$ for all $w \in s$, $M, \sigma_a(w) \models \varphi$
- $M, s \models E_a\varphi \iff$ for all $w \in s$ and $t \in \Sigma_a(w)$, $M, t \models \varphi$

Remark

All facts discussed before for InqD_π extend straghtforwardly to IEL.

Inquisitive epistemic logic

Knowledge modality

$M, w \models K_a\varphi \iff M, \sigma_a(w) \models \varphi$

Knowing a declarative

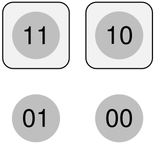
$M, w \models K_a\alpha \iff \alpha$ is established in $\sigma_a(w)$

$M, w \models K_a\alpha \iff M, v \models \alpha$ for all $v \in \sigma_a(w)$

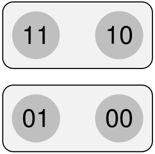
Knowing an interrogative

$M, w \models K_a\mu \iff \mu$ is resolved in $\sigma_a(w)$

Ex. $K_a?p \equiv K_a p \vee K_a\neg p$



$\Sigma_a(w)$



$?p$

Inquisitive epistemic logic

Entertain modality

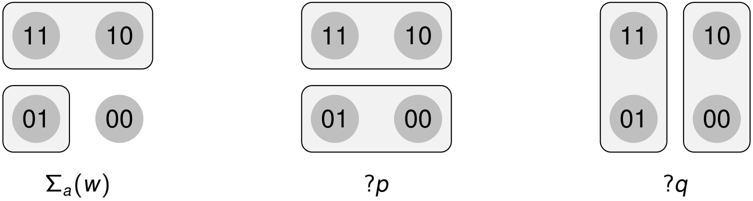
$M, w \models E_a\varphi \iff M, t \models \varphi$ for all $t \in \Sigma_a(w)$

Entertaining a declarative

$M, w \models E_a\alpha \iff M, w \models K_a\alpha$

Entertaining an interrogative

$M, w \models E_a\mu \iff \mu$ is resolved in states where a's issues are resolved



Inquisitive epistemic logic

Definition (Entailment)

$\Phi \models \psi \iff$ for any IEL-model M and state s, if $M, s \models \Phi$ then $M, s \models \psi$

Axiomatization

Expanding the derivation system for InqD_π with a few standard axioms and rules for the modalities, we get a complete axiomatization of IEL.

Two remarks

1. The logic for declaratives is not autonomous: reasoning with interrogative is crucial in drawing declarative inferences.

   Ex: $E_a\mu \models E_a\nu \iff \mu \models \nu$

2. The logical properties of the modalities turn out to be more general than their Kripkean framework from which they usually arise.

Conclusions

- We have seen ~~three~~ two combined logics of information and issues.
- InqD$_\pi$ extends classical propositional logic to reason with issues.

  Ex. $p \leftrightarrow q \wedge r,\ ?q \wedge ?r \models ?p$
- IEL extends epistemic logic to reason about entertaining issues.

  Ex. $K_a(p \leftrightarrow q \wedge r),\ K_a q \models E_a?p \rightarrow E_a?r$
- IDEL extends PAL to reason about raising issues.

  Ex. $K_a(p \leftrightarrow q \wedge r),\ K_a q \models [?p]E_a?r$


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
BÏYAN
SHUKRIA
TINGKI
YAQHANYELAY
TASHAKKUR ATU
SUKSAMA
EKHMET
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
KOMAPSUMNIDA
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