Suppositional inquisitive semantics

Jeroen Groenendijk and Floris Roelofsen
partly based on joint work with Martin Aher

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Overview

1. Suppositional inquisitive semantics

1.1. Basic motivation: support, reject, dismiss

Support

- Inquisitive semantics takes sentences to express a proposal to update the common ground of the conversation (CG) in one or more ways.

- The question in (1a) proposes two alternative ways to update the CG, which correspond to the two responses (1b-c).

(1) a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
    b. If Alf goes, then Bea will go as well. $p \rightarrow q$
    c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$

- Basic inquisitive semantics (InqB) accounts for the intuition that (1b-c) are responses that, *if accepted by the other conversational participants*, yield a CG that supports the question in (1a), settling the proposal that it expresses.

Support and reject

- InqB does not account for the intuition that (1c) rejects the proposal expressed by (1b), and vice versa.

(1) a. If Alf goes to the party, will Bea go too? $p \to ?q$
b. If Alf goes, then Bea will go as well. $p \to q$
c. If Alf goes, then Bea will not go. $p \to \neg q$

- Radical inquisitive semantics (InqR) does account for this.

- It achieves this by not only specifying support-conditions, as InqB does, but simultaneously also rejection-conditions.

Support, reject, dismiss

- InqB and InqR do not account for the intuition that (1d) dismisses a supposition that is shared by (1a)-(1c).

(1) a.	If Alf goes to the party, will Bea go too?	$p \rightarrow ?q$
b.	If Alf goes, then Bea will go as well.	$p \rightarrow q$
c.	If Alf goes, then Bea will not go.	$p \rightarrow \neg q$
d.	Alf will not go to the party.	$\neg p$

- This is just as much a way of settling the proposals that these sentences express, on a par with support and rejection.
- Suppositional inq semantics (InqS) aims to characterize when a response suppositionally dismisses a given proposal.
- To achieve this, it does not only specify conditions for support and rejection, but also for supposition dismissal.

Reject and dismiss

- in InqR $\neg p$ both supports and rejects $p \to q$.
- Couldn't *that* mean that $\neg p$ suppositionally dismisses $p \to q$?
- This does not work for slightly more complex examples:

(2)	a.	If Alf or Cor goes, Bea will go too.	$(p \vee q) \to r$
	b.	Alf will not go.	$\neg p$
	c.	And if Cor goes, then Bea will not go.	$q \to \neg r$

- Intuitively, (2c) rejects (2a), but (2b) does not reject it, but dismisses a supposition of (2a).
- In InqR (2b) does reject (2a), but does not support it.
- Taking: suppositional dismissal = support + rejection, does not account for the fact that (2b) dismisses a supposition of (2a).
- InqS accounts for this, plus for that once (2b) is *accepted*, (2a) is no longer supportable, but is still rejectable, as (2c) shows.

1.2. Basic semantic notions

Some basic notions

- We consider a language $\mathcal{L}$ of propositional logic.
- We let $?\varphi$ be an abbreviation of $\varphi \vee \neg\varphi$
- Sentences are evaluated relative to information states.
- An information state *s* is set of possible worlds.
- A possible world *w* is a valuation function that assigns the value 1 or 0 to each atomic sentence in $\mathcal{L}$.
- We use ω to denote the set of all worlds, the ignorant state.
- We refer to the empty set as the absurd or inconsistent state.

Global structure of the semantics

- The semantics for $\mathcal{L}$ is given by a simultaneous recursive definition of three basic semantic relations:

  1. $s \models^{+} \varphi$ state s supports φ InqB
  2. $s \models^{-} \varphi$ state s rejects φ InqR
  3. $s \models^{\circ} \varphi$ state s dismisses a supposition of φ InqS

- By $[\varphi]^{+}$ we denote $\{s \subseteq \omega \mid s \models^{+} \varphi\}$, similarly for $[\varphi]^{-}$ and $[\varphi]^{\circ}$

- The proposition expressed by φ, $[\varphi]$, is determined by:

$$[\varphi] = \langle [\varphi]^{+}, [\varphi]^{-}, [\varphi]^{\circ} \rangle$$

Notation convention for representing states

- Let $|\varphi|$ denote the set of worlds where φ is classically true
- This gives us a convenient notation for states. For instance:

$$
\begin{array}{lll}
|p| & \models^{+} & p \vee q \\
|\neg p| & \models^{-} & p \wedge q \\
|\neg p| & \models^{\circ} & p \rightarrow q
\end{array}
$$

1.3. Suppositional inquisitive meaning postulates

Downward closure / persistence

- A distinctive feature of InqB is that $[\varphi]^+$ is downward closed
  - If $s \models^+ \varphi$, then for any $t \subseteq s$: $t \models^+ \varphi$

  That is, in InqB support is persistent

- In InqR, both $[\varphi]^+$ and $[\varphi]^-$ are downward closed
  - If $s \models^+ \varphi$, then for any $t \subseteq s$: $t \models^+ \varphi$
  - If $s \models^- \varphi$, then for any $t \subseteq s$: $t \models^- \varphi$

  That is, in InqR both support and rejection are persistent

- Underlying idea: if s supports/rejects a sentence φ, then any more informed state $t \subseteq s$ will support/reject φ as well

- Information growth cannot lead to retraction of support/reject

Persistence and suppositional dismissal

- As soon as we take suppositional dismissal into account this central idea from InqB and InqR is no longer defensible
- For instance, we want that:

$$|p \rightarrow q| \models^{+} p \rightarrow q$$

  But we also want that:

$$\begin{array}{lll} |\neg p| & \models^{\circ} & p \rightarrow q \\ |\neg p| & \not\models^{+} & p \rightarrow q \end{array}$$

- So: information growth can lead to suppositional dismissal, and thereby to retraction of support (or retraction of rejection)

Persistence modulo suppositional dismissal

- Fortunately, there is a natural way to adapt the idea that support and rejection are persistent to the setting of InqS
- Namely, in InqS we postulate that support and rejection are persistent modulo dismissal of a supposition, and that dismissal itself is fully persistent:
  - If $s \models^{+} \varphi$ and $t \subseteq s$, then $t \models^{+} \varphi$ or $t \models^{\circ} \varphi$
  - If $s \models^{-} \varphi$ and $t \subseteq s$, then $t \models^{-} \varphi$ or $t \models^{\circ} \varphi$
  - If $s \models^{\circ} \varphi$ and $t \subseteq s$, then $t \models^{\circ} \varphi$

Two more postulates

Second postulate

- The inconsistent state suppositionally dismisses any sentence φ, and never supports or rejects it. That is, for any φ:

$$\emptyset \models^{\circ} \varphi$$
$$\emptyset \not\models^{+} \varphi$$
$$\emptyset \not\models^{-} \varphi$$

Third postulate

- Support and rejection are mutually exclusive : $[\varphi]^{+} \cap [\varphi]^{-} = \emptyset$
- The postulates do not exclude that for some φ and $s \neq \emptyset$:
  - $s \models^{+} \varphi$ and $s \models^{\circ} \varphi$
  - $s \models^{-} \varphi$ and $s \models^{\circ} \varphi$

Finally

- Final postulate: any completely informed consistent state $\{w\}$ supports, rejects, or suppositionally dismisses any sentence:

$$\forall \varphi \in \mathcal{L} : \forall w \in \omega : \{w\} \in ([\varphi]^{+} \cup [\varphi]^{-} \cup [\varphi]^{\circ})$$

Propositions as conversational issues

- The postulates imply that the three components of a proposition jointly form a non-empty downward closed set of states that cover the set of all worlds:

$$\bigcup([\varphi]^{+} \cup [\varphi]^{-} \cup [\varphi]^{\circ}) = \omega$$

- In terms of InqB, InqS propositions are issues over ω.
- The issue embodied by $[\varphi]$ is a conversational issue, it specifies several appropriate ways of responding to φ.

1.4. Some semantical and logical notions

Three core semantic notions

Informative content of a sentence

- $\text{info}(\varphi) := \bigcup[\varphi]^+$

Informative, inquisitive and suppositional sentences

- φ is informative iff $\text{info}(\varphi) \neq \omega$
- φ is inquisitive iff $[\varphi]^+ \neq \emptyset$ and $\text{info}(\varphi) \not\models^+ \varphi$
- φ is suppositional iff $[\varphi]^\circ \neq \{\emptyset\}$

- A sentence φ suppositional iff there is at least one consistent state s such that $s \models^\circ \varphi$

Alternatives and inquisitiveness

Support and reverse alternatives for a sentence

- $\textsc{alt}^{+}(\varphi) := \{s \mid s \models^{+} \varphi$ and there is no $t \supset s$ such that $t \models^{+} \varphi\}$
- $\textsc{alt}^{-}(\varphi) := \{s \mid s \models^{-} \varphi$ and there is no $t \supset s$ such that $t \models^{-} \varphi\}$

Alternatives and inquisitiveness in a finite setting

- φ is support inquisitive iff $\textsc{alt}^{+}(\varphi)$ has two or more elements
- φ is reverse inquisitive iff $\textsc{alt}^{-}(\varphi)$ has two or more elements

Some derived semantic relations

- In terms of the three basic semantic relations, we can define other ones, such as:

Excluding supportability and rejectability

- $s \models_{\lightning} \varphi$ iff $\forall t \subseteq s$: $t \not\models^{+} \varphi$
- $s \models_{\surd} \varphi$ iff $\forall t \subseteq s$: $t \not\models^{-} \varphi$

- $s \models_{\bullet} \varphi$ iff $s \models_{\lightning} \varphi$ and $s \models_{\surd} \varphi$
- $s \models_{\diamond} \varphi$ iff $s \not\models_{\lightning} \varphi$ and $s \not\models_{\surd} \varphi$

Some more derived semantic relations

Indefeasible and defeasible support and rejection

- $s \models^{+}_{\surd} \varphi$ iff $s \models^{+} \varphi$ and $s \models_{\surd} \varphi$
- $s \models^{+}_{\diamond} \varphi$ iff $s \models^{+} \varphi$ and $s \not\models_{\surd} \varphi$

- $s \models^{-}_{\lightning} \varphi$ iff $s \models^{-} \varphi$ and $s \models_{\lightning} \varphi$
- $s \models^{-}_{\diamond} \varphi$ iff $s \models^{-} \varphi$ and $s \not\models_{\lightning} \varphi$

Dismissing supportability and rejectability

- $s \models^{\circ}_{\lightning} \varphi$ iff $s \models^{\circ} \varphi$ and $s \models_{\lightning} \varphi$ and $s \not\models^{-} \varphi$
- $s \models^{\circ}_{\surd} \varphi$ iff $s \models^{\circ} \varphi$ and $s \models_{\surd} \varphi$ and $s \not\models^{+} \varphi$

Defeasibility in InqS

- The distinction between defeasible and indefeasible support / rejection will only start playing a role once we add epistemic modalities to the language
- In InqS support and rejection of implication is indefeasible
- *might*-sentences will be support-defeasible, and rejection-indefeasible
- *must*-sentences will be rejection-defeasible, and support-indefeasible

Responsehood relations

- We can define a range of logical responsehood relations according to the following scheme, filling in different semantic relations for $\models^{\dagger}$ (we restrict ourselves here to non-inquisitive responses):
  - For any $\varphi, \psi \in \mathcal{L}$, where φ is non-inquisitive:

$$\psi \models^{\dagger} \varphi \text{ iff } \forall s: \text{ if } s \models^{+} \psi, \text{ then } s \models^{\dagger} \varphi$$

- Suppositional inquisitive logic is a logic of responsehood

1.5. Statement of the semantics

Atomic sentences

- $s \models^{+} p$ iff $s \neq \emptyset$ and $\forall w \in s\colon w(p) = 1$
  $s \models^{-} p$ iff $s \neq \emptyset$ and $\forall w \in s\colon w(p) = 0$
  $s \models^{\circ} p$ iff $s = \emptyset$

- Atomic sentences are not suppositional, since only the inconsistent state can dismiss a supposition of *p*.
- Atomic sentences are not inquisitive, since there is only a single support-alternative and a single reject-alternative:

$$\begin{aligned} \textsc{alt}^{+}(p) &= \{|p|\} \\ \textsc{alt}^{-}(p) &= \{|\neg p|\} \end{aligned}$$

Negation

$$s \models^{+} \neg\varphi \quad \text{iff} \quad s \models^{-} \varphi$$
$$s \models^{-} \neg\varphi \quad \text{iff} \quad s \models^{+} \varphi$$
$$s \models^{\circ} \neg\varphi \quad \text{iff} \quad s \models^{\circ} \varphi$$

- The suppositional content of φ is inherited by its negation $\neg\varphi$
- Unlike in InqB: $\neg\neg\varphi \equiv \varphi$

Disjunction

- $s \models^+ \varphi \vee \psi$ iff $s \models^+ \varphi$ or $s \models^+ \psi$
  $s \models^- \varphi \vee \psi$ iff $s \models^- \varphi$ and $s \models^- \psi$
  $s \models^\circ \varphi \vee \psi$ iff $s \models^\circ \varphi$ or $s \models^\circ \psi$

- The suppositional content of φ and ψ is inherited by the disjunction $\varphi \vee \psi$

- The disjunction $p \vee q$ is support-inquisitive: there are two support-alternatives for $p \vee q$:

$$\textsc{alt}^+(p \vee q) = \{|p|, |q|\}$$

Conjunction

- $s \models^+ \varphi \wedge \psi$ iff $s \models^+ \varphi$ and $s \models^+ \psi$

  $s \models^- \varphi \wedge \psi$ iff $s \models^- \varphi$ or $s \models^- \psi$

  $s \models^\circ \varphi \wedge \psi$ iff $s \models^\circ \varphi$ or $s \models^\circ \psi$

- The suppositional content of φ and ψ is inherited by the conjunction $\varphi \wedge \psi$
- The conjunction $p \wedge q$ is reverse inquisitive: there are two reverse alternatives for $p \wedge q$:

$$\text{ALT}^-(p \wedge q) = \{|\neg p|, |\neg q|\}$$

Triggering and projection of suppositional content

- None of the clauses in the semantics for the Boolean fragment of the language has the potential to trigger suppositional content.
- Atomic sentences are not suppositional, and negation, disjunction and conjunction only project suppositional content of their subformulas in a cumulative way.
- For the language at hand, implication is the only trigger of suppositional content.
- Implication also projects the suppositional content of its consequent, but relativized to its antecedent.

Supposition triggered by implication

- The supposition that is triggered by an implication concerns the supposability of its antecedent.
- The supposability of a sentence is determined by:
  - (a) the existence of support-alternatives for it.
  - (b) the supposability of its support-alternatives.
- Suppositional dismissal of an implication occurs in *s*, when there is no support-alternative for its antecedent, or when there is some support-alternative that is not supposable in *s*.

Supporting an implication: InqB versus InqS

- The clause for implication in InqB is as follows:

  $s \models \varphi \to \psi$ iff $\forall t$: if $t \models \varphi$, then $t \cap s \models \psi$

- We can also formulate this in terms of the alternatives for φ:

  $s \models \varphi \to \psi$ iff $\forall \alpha \in \text{ALT}[\varphi] : \alpha \cap s \models \psi$

- Since in InqB support is fully persistent, it makes no difference whether we consider just the support-alternatives for φ or all states that support it.

- In InqS, where support is only persistent modulo suppositional dismissal, it does potentially make a difference.

- We should only consider the support-alternatives for φ, because other states that support φ may contain additional information which causes suppositional dismissal of ψ.

- This should not be a reason for support of $\varphi \to \psi$ to fail.

Implication in InqS: the intuitive idea

- *s* supports $\varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and for every $\alpha \in \textsc{alt}[\varphi]^+$:

  (a) α is supposable in *s*, and

  (b) $\alpha \cap s$ supports ψ

- *s* rejects $\varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and for some $\alpha \in \textsc{alt}[\varphi]^+$:

  (a) α is supposable in *s*, and

  (b) $\alpha \cap s$ rejects ψ

- *s* dismisses $\varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$, or for some $\alpha \in \textsc{alt}[\varphi]^+$:

  (a) α is is not supposable in *s*, or

  (b) $\alpha \cap s$ dismisses a supposition of ψ

Supposability of support alternatives: the intuitive idea

- Recall the meaning postulates, in particular: persistence of support modulo dismissal of a supposition
- When α is a support alternative for φ, then there are two cases in which we take α not to be supposable in *s*

Two cases where supposability should not hold

(a) α supports φ, but $\alpha \cap s$ no longer supports φ, whence it must dismiss a supposition of φ

This is bound to be the case if, but not only if, $\alpha \cap s = \emptyset$

(b) Both α and $\alpha \cap s$ support φ, but there is some state *t* in between α and $\alpha \cap s$ that does not support φ, whence *t*, and also $\alpha \cap s$, must dismiss a supposition of φ

Supposability of support alternatives

- A support alternative $\alpha \in \textsc{alt}^+(\varphi)$ is supposable in s, $s \lhd \alpha$ iff
  - (a) $\alpha \cap s \models^+ \varphi$ which implies $\alpha \cap s \neq \emptyset$
  - (b) For all t such that $\alpha \supset t \supset (\alpha \cap s)$: $t \models^+ \varphi$

Supposability and support-convexity

- φ is support-convex iff

  $$\forall s, t, u: \text{ if } s \subset t \subset u \text{ and } s, u \in [\varphi]^+, \text{ then } t \in [\varphi]^+$$

- If φ is support-convex, then clause (b) can be ignored
- Example: $\varphi = (p \to q) \vee r$ is not support-convex:

  $|p \to q| \models^+ \varphi$, and $|\neg p| \not\models^+ \varphi$, but $|\neg p \wedge r| \models^+ \varphi$

- $|p \to q|$ is a support-alternative for φ not supposable in $s = |\neg p \wedge r|$

Supposability of support alternatives

- A support alternative $\alpha \in \text{ALT}^{+}(\varphi)$ is supposable in s, $s \lhd \alpha$ iff
  - (a) $\alpha \cap s \models^{+} \varphi$ which implies $\alpha \cap s \neq \emptyset$
  - (b) For all t such that $\alpha \supset t \supset (\alpha \cap s)$: $t \models^{+} \varphi$

Supposability and support-density

- φ is support-dense iff

  $$\forall s, t : \text{if } s \in [\varphi]^{+} \text{ and } t \subset s \text{ and } t \neq \emptyset, \text{ then } t \in [\varphi]^{+}$$

- Support-dense implies support-convex: clause (b) void
- If φ is support-dense, then clause (a) reduces to $\alpha \cap s \neq \emptyset$
- Not suppositional implies support-dense, so the entire Boolean fragment of the language is support-dense
- Suppositionality does not imply non-density, for instance:

  $\neg p \vee (p \rightarrow q)$ is suppositional and dense as well

Implication in InqS spelled out

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}^{+}(\varphi) \neq \emptyset$ and

  $\forall \alpha \in \textsc{alt}^{+}(\varphi)$: $s \lhd \alpha$ and $\alpha \cap s \models^{+} \psi$

- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}^{+}(\varphi) \neq \emptyset$ and

  $\exists \alpha \in \textsc{alt}^{+}(\varphi)$: $s \lhd \alpha$ and $\alpha \cap s \models^{-} \psi$

- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}^{+}(\varphi) = \emptyset$ or

  $\exists \alpha \in \textsc{alt}^{+}(\varphi)$: $s \not\lhd \alpha$ or $\alpha \cap s \models^{\circ} \psi$

Reductions

- If φ is support-convex: $s \lhd \alpha \rightsquigarrow \alpha \cap s \models^{+} \varphi$
- If φ is support-dense: $s \lhd \alpha \rightsquigarrow \alpha \cap s \neq \emptyset$
- If ψ is support-dense: $\alpha \cap s \models^{\circ} \psi \rightsquigarrow \alpha \cap s = \emptyset$

Implication, reduction for non-inquisitive antecedent

- $s \models^{+} \varphi \rightarrow \psi$ iff $\text{info}(\varphi) \neq \emptyset$ and

  $s \lhd \text{info}(\varphi)$ and $\text{info}(\varphi) \cap s \models^{+} \psi$

- $s \models^{-} \varphi \rightarrow \psi$ iff $\text{info}(\varphi) \neq \emptyset$ and

  $s \lhd \text{info}(\varphi)$ and $\text{info}(\varphi) \cap s \models^{-} \psi$

- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\text{info}(\varphi) = \emptyset$ or

  $s \ntriangleleft \text{info}(\varphi)$ or $\text{info}(\varphi) \cap s \models^{\circ} \psi$

Further reductions

- If φ is support-convex: $s \lhd \text{info}(\varphi) \rightsquigarrow \text{info}(\varphi) \cap s \models^{+} \varphi$
- If φ is support-dense: $s \lhd \text{info}(\varphi) \rightsquigarrow \text{info}(\varphi) \cap s \neq \emptyset$
- If ψ is support-dense: $\text{info}(\varphi) \cap s \models^{\circ} \psi \rightsquigarrow \text{info}(\varphi) \cap s = \emptyset$

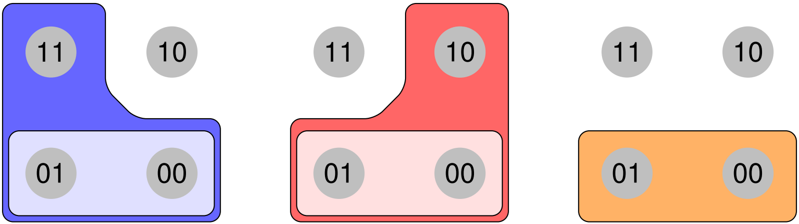
1.5. Examples

Our initial example: $p \to q$

$$s \models^{+} p \to q \quad \text{iff} \quad s \cap |p| \models^{+} q$$

$$s \models^{-} p \to q \quad \text{iff} \quad s \cap |p| \models^{-} q$$

$$s \models^{\circ} p \to q \quad \text{iff} \quad s \cap |p| = \emptyset$$



(a) support (b) reject (c) dismiss

How to read the pictures

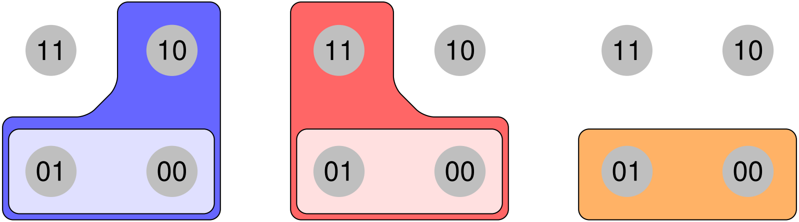
- Support is persistent modulo suppositional dismissal.
  - We depict maximal states that support φ, and if necessary also the maximal substates of these states that no longer support φ.
  - We think of these substates as support holes.
- Rejection is persistent modulo suppositional dismissal.
  - We depict maximal states that reject φ, and if necessary also the maximal substates of these states that no longer reject φ.
  - We think of these substates as rejection holes.
- Dismissal is fully persistent.
  - We depict only maximal states that dismiss a supposition of φ.
  - All substates thereof also dismiss a supposition of φ.

Our initial example: $p \to \neg q$

$$s \models^{+} p \to \neg q \quad \text{iff} \quad s \cap |p| \models^{+} \neg q$$

$$s \models^{-} p \to \neg q \quad \text{iff} \quad s \cap |p| \models^{-} \neg q$$

$$s \models^{\circ} p \to \neg q \quad \text{iff} \quad s \cap |p| = \emptyset$$



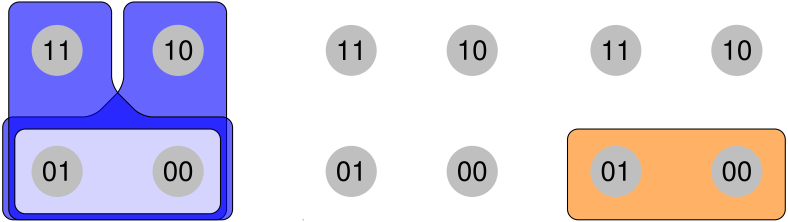
(a) support (b) reject (c) dismiss

Our initial example: $p \rightarrow ?q$

$s \models^{+} p \rightarrow ?q$ iff $s \cap |p| \models^{+} q$ or $s \cap |p| \models^{+} \neg q$

$s \models^{-} p \rightarrow ?q$ iff $s \cap |p| \models^{-} q$ and $s \cap |p| \models^{-} \neg q$ impossible

$s \models^{\circ} p \rightarrow ?q$ iff $s \cap |p| = \emptyset$



(a) support (b) reject (c) dismiss

Desired predictions

(1)
a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
b. If Alf goes, then Bea will go as well. $p \rightarrow q$
c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$
d. Alf won't go. $\neg p$

- Both (1b) and (1c) support the conditional question in (1a):

$$p \rightarrow q \quad \models^{+} \quad p \rightarrow ?q$$

$$p \rightarrow \neg q \quad \models^{+} \quad p \rightarrow ?q$$

- (1b) and (1c) are contradictory, they reject each other:

$$p \rightarrow q \quad \models^{-} \quad p \rightarrow \neg q$$

$$p \rightarrow \neg q \quad \models^{-} \quad p \rightarrow q$$

Desired predictions

(1) a. If Alf goes to the party, will Bea go too? $p \to ?q$
b. If Alf goes, then Bea will go as well. $p \to q$
c. If Alf goes, then Bea will not go. $p \to \neg q$
d. Alf won't go. $\neg p$

- Finally, (1d) suppositionally dismisses (1a)-(1c) :

$$\neg p \models^{\circ} p \to ?q$$

$$\neg p \models^{\circ} p \to q$$

$$\neg p \models^{\circ} p \to \neg q$$

- In particular:

$$\neg p \not\models^{+} p \to q$$

Additional prediction, whether desired or not

(3) a. If Alf goes to the party, will Bea go too? $p \to ?q$
b. ?Bea will go to the party. q
c. Whether Alf goes or not, Bea will go. $(p \vee \neg p) \to q$
d. If Alf goes, Bea will not go. $p \to \neg q$

- The response in (2b) needs marking, (2c) is fine.
- (2c) and (2d) are contradictory responses to (2a).
- We will return to the example later. For now we note:

$$q \not\models^{+} p \to ?q$$

$$q \not\models^{+} p \to q$$

- Reason: $|q \wedge \neg p|$ is a state that supports q, but it suppositionally dismisses, and therefore does not support $p \to q$ and $p \to ?q$.

Three more complex examples

We will consider three more complex examples:

(1) Inquisitive antecedent: $(p \vee q) \to r$

(2) Suppositional consequent: $p \to (q \to r)$

(3) Suppositional antecedent: $(p \to q) \to r$

Case 1: inquisitive antecedent: $(p \vee q) \rightarrow r$

- Both antecedent and consequent are not suppositional, and hence dense
- There are two support alternatives for the antecedent:

$$\textsc{alt}^+(p \vee q) = \{|p|, |q|\}$$

- So we have:

$$s \models^+ (p \vee q) \rightarrow r \quad \text{iff} \quad \forall \alpha \in \{|p|, |q|\} : \alpha \cap s \models^+ r$$

$$s \models^- (p \vee q) \rightarrow r \quad \text{iff} \quad \exists \alpha \in \{|p|, |q|\} : \alpha \cap s \models^- r$$

$$s \models^\circ (p \vee q) \rightarrow r \quad \text{iff} \quad \exists \alpha \in \{|p|, |q|\} : \alpha \cap s = \emptyset$$

Case 1: inquisitive antecedent: $(p \vee q) \rightarrow r$

$$s \models^{+} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s \models^{+} r \ \text{ and } \ |q| \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s \models^{-} r \ \text{ or } \ |q| \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s = \emptyset \ \text{ or } \ |q| \cap s = \emptyset$$

- Some (non-)supporting responses:

$$(p \rightarrow r) \wedge (q \rightarrow r) \quad \models^{+} \quad (p \vee q) \rightarrow r$$

$$\neg p \wedge \neg q \quad \not\models^{+} \quad (p \vee q) \rightarrow r$$

Case 1: inquisitive antecedent: $(p \vee q) \rightarrow r$

$$s \models^{+} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s \models^{+} r \;\text{ and }\; |q| \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s \models^{-} r \;\text{ or }\; |q| \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s = \emptyset \;\text{ or }\; |q| \cap s = \emptyset$$

- Some rejecting responses:

$$p \rightarrow \neg r \quad \models^{-} \quad (p \vee q) \rightarrow r$$

$$\neg p \wedge (q \rightarrow \neg r) \quad \models^{\ominus} \quad (p \vee q) \rightarrow r$$

Case 1: inquisitive antecedent: $(p \vee q) \rightarrow r$

$$s \models^{+} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s \models^{+} r \;\text{ and }\; |q| \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s \models^{-} r \;\text{ or }\; |q| \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \rightarrow r \quad \text{iff} \quad |p| \cap s = \emptyset \;\text{ or }\; |q| \cap s = \emptyset$$

- Some responses that dismiss a supposition:

$$\neg p \quad \models^{\circ}_{\lightning} \quad (p \vee q) \rightarrow r$$

$$\neg p \wedge \neg q \quad \models^{\circ}_{\bullet} \quad (p \vee q) \rightarrow r$$

Affirming the consequent again

(3) a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
b. Whether Alf goes or not, Bea will go. $(p \vee \neg p) \rightarrow q$
c. If Alf goes, Bea will not go. $p \rightarrow \neg q$

- (3b) is a felicitous, supporting response to (3a).
- (3b) and (3c) are contradictory responses.

$$(p \vee \neg p) \rightarrow q \models^{+} p \rightarrow ?q$$

$$p \rightarrow \neg q \models^{-} (p \vee \neg p) \rightarrow q$$

$$(p \vee \neg p) \rightarrow q \models^{-} p \rightarrow \neg q$$

$$(p \vee \neg p) \rightarrow q \models^{+} q$$

Case 2: suppositional consequent: $p \to (q \to r)$

- The antecedent is still non-suppositional and hence support-dense

- Moreover, there is a single support-alternative for the antecedent:

$$\textsc{alt}^+(p) = \{|p|\}$$

- So we have:

$$s \models^+ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^+ q \to r$$

$$s \models^- p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^- q \to r$$

$$s \models^\circ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^\circ q \to r$$

Case 2: suppositional consequent: $p \rightarrow (q \rightarrow r)$

$$s \models^+ p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \models^+ q \rightarrow r$$

$$s \models^- p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \models^- q \rightarrow r$$

$$s \models^\circ p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \models^\circ q \rightarrow r$$

- Since the consequent is a simple conditional, this can be further reduced to:

$$s \models^+ p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^+ r$$

$$s \models^- p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^- r$$

$$s \models^\circ p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

Case 2: suppositional consequent: $p \to (q \to r)$

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some (non-)supporting responses:

$$(p \wedge q) \to r \quad \models^{+} \quad p \to (q \to r)$$

$$\neg p \quad \not\models^{+} \quad p \to (q \to r)$$

$$\neg q \quad \not\models^{+} \quad p \to (q \to r)$$

Case 2: suppositional consequent: $p \to (q \to r)$

$$s \models^+ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^+ r$$

$$s \models^- p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^- r$$

$$s \models^\circ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some (non-)rejecting responses:

$$(p \wedge q) \to \neg r \quad \models^- \quad p \to (q \to r)$$

$$p \to \neg r \quad \not\models^- \quad p \to (q \to r)$$

$$p \to ((q \vee \neg q) \to \neg r) \quad \models^- \quad p \to (q \to r)$$

$$q \to \neg r \quad \not\models^- \quad p \to (q \to r)$$

$$(p \vee \neg p) \to (q \to \neg r) \quad \models^- \quad p \to (q \to r)$$

Case 2: suppositional consequent: $p \to (q \to r)$

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some responses that dismiss a supposition:

$$\neg p \quad \models^{\circ} \quad p \to (q \to r)$$

$$\neg q \quad \models^{\circ} \quad p \to (q \to r)$$

Case 3: suppositional antecedent: $(p \to q) \to r$

- Now the antecedent is suppositional and not support-dense, but it is support-convex
- There is a single support-alternative α for the antecedent:

$$\alpha = |p \to q|$$

- So we have:

$$s \models^+ (p \to q) \to r \text{ iff } s \cap |p \to q| \models^+ p \to q \qquad = s \not\subseteq |\neg p|$$
$$\text{and } s \cap |p \to q| \models^+ r$$

$$s \models^- (p \to q) \to r \text{ iff } s \cap |p \to q| \models^+ p \to q \qquad = s \not\subseteq |\neg p|$$
$$\text{and } s \cap |p \to q| \models^- r$$

$$s \models^\circ (p \to q) \to r \text{ iff } s \cap |p \to q| \not\models^+ p \to q \qquad = s \subseteq |\neg p|$$
$$\text{or } s \cap |p \to q| = \emptyset$$

Case 3: suppositional antecedent: $(p \to q) \to r$

$$s \models^{+} (p \to q) \to r \text{ iff } s \not\subseteq |\neg p| \text{ and } s \cap |p \to q| \models^{+} r$$

$$s \models^{-} (p \to q) \to r \text{ iff } s \not\subseteq |\neg p| \text{ and } s \cap |p \to q| \models^{-} r$$

$$s \models^{\circ} (p \to q) \to r \text{ iff } s \subseteq |\neg p|$$

- Some non-supporting responses:

$$r \quad \not\models^{+} \quad (p \to q) \to r$$

$$\neg p \quad \not\models^{+} \quad (p \to q) \to r$$

$$p \wedge \neg q \quad \not\models^{+} \quad (p \to q) \to r$$

$$p \to \neg q \quad \not\models^{+} \quad (p \to q) \to r$$

Case 3: suppositional antecedent: $(p \rightarrow q) \rightarrow r$

$$s \models^{+} (p \rightarrow q) \rightarrow r \text{ iff } s \not\subseteq |\neg p| \text{ and } s \cap |p \rightarrow q| \models^{+} r$$

$$s \models^{-} (p \rightarrow q) \rightarrow r \text{ iff } s \not\subseteq |\neg p| \text{ and } s \cap |p \rightarrow q| \models^{-} r$$

$$s \models^{\circ} (p \rightarrow q) \rightarrow r \text{ iff } s \subseteq |\neg p|$$

- Some rejecting responses:

$$(p \rightarrow q) \rightarrow \neg r \quad \models^{-} \quad (p \rightarrow q) \rightarrow r$$

$$p \wedge (q \rightarrow \neg r) \quad \models^{-} \quad (p \rightarrow q) \rightarrow r$$

Case 3: suppositional antecedent: $(p \to q) \to r$

$$s \models^{+} (p \to q) \to r \text{ iff } s \not\subseteq |\neg p| \text{ and } s \cap |p \to q| \models^{+} r$$

$$s \models^{-} (p \to q) \to r \text{ iff } s \not\subseteq |\neg p| \text{ and } s \cap |p \to q| \models^{-} r$$

$$s \models^{\circ} (p \to q) \to r \text{ iff } s \subseteq |\neg p|$$

- Some responses that dismiss a supposition:

$$\neg p \models^{\circ} (p \to q) \to r$$

$$p \to \neg q \models^{\circ} (p \to q) \to r$$

$$p \wedge \neg q \models^{\circ} (p \to q) \to r$$

Conclusion first part

- The general perspective on meaning in inquisitive semantics is that sentences express proposals to update the CG in one or more ways
- There are several ways one may respond to such proposals, depending on one's information state
- InqB characterizes which states support a given proposal
- InqR also characterizes which states reject a given proposal
- InqS further distinguishes states that dismiss a supposition of a given proposal
- We thus arrive at a more and more fine-grained formal characterization of proposals, and thereby a more and more fine-grained characterization of meaning

Conclusion first part

- This in turn leads to a better account of the behavior of certain types of sentences in conversation
- InqS especially improves on InqB and InqR in its treatment of conditional statements and questions
- Paradigm example:

$$p \to q \quad \text{evaluated in the state} \quad |\neg p|$$

- InqB: support
- InqR: both support and reject
- InqS: suppositional dismissal

2. Suppositional epistemic *might* and *must*

2.1. Epistemic *might* as a supposability check

Suppositional epistemic *might*

Might as a supposability check

- In InqS, $\Diamond\varphi$ can be treated as inducing a supposability check.
- In the most basic cases, checking supposability amounts to checking consistency.
- Thus, in these basic cases, our analysis of $\Diamond\varphi$ comes down to Veltman's analysis of *might* in update semantics (US).
- However, for more involved cases, the two analyses diverge.

Persistence

- For Veltman, $\Diamond\varphi$ is a basic example of a non-persistent update.
- In InqS, both $\Diamond\varphi$ and $\Box\varphi$ are support / reject-persistent modulo suppositional dismissal.

Reminder

Suppositionally dismissing supportability

- $s \models^{\circ}_{\lightning} \varphi$ iff $s \models^{\circ} \varphi$ and $s \not\models^{-} \varphi$ and $\forall t \subseteq s\colon\ t \not\models^{+} \varphi$.

For dense (non-suppositional) φ

- $s \models^{\circ}_{\lightning} \varphi$ iff $s = \emptyset$.

Generally

- If $s \models^{\circ}_{\lightning} \varphi$, then no support-alternative for φ is supposable in s.

Suppositional *might*: the intuitive idea

$\Diamond\varphi$ expresses a proposal to check the supposability of φ in s

- s supports $\Diamond\varphi$ iff
  - (a) there is at least one support-alternative for φ and
  - (b) every support-alternative for φ is supposable in s
- s rejects $\Diamond\varphi$ iff
  - (a) s does not suppositionally dismiss supportability of φ and
  - (b) every support-alternative for φ is not supposable in s
- s dismisses a supposition of $\Diamond\varphi$ iff
  - (a) there is no support-alternative for φ or
  - (b) some support-alternative for φ is not supposable in s

Suppositional *might*: support and dismissal

Support and dismissing a supposition contradict each other

- s supports $\Diamond\varphi$ iff
  - (a) there is at least one support-alternative for φ and
  - (b) every support-alternative for φ is supposable in s
- s dismisses a supposition of $\Diamond\varphi$ iff
  - (a) there is no support-alternative for φ or
  - (b) some support-alternative for φ is not supposable in s

Suppositional *might*: rejection and dismissal

Rejection implies suppositional dismissal

- *s* rejects $\Diamond\varphi$ iff
  (a) *s* does not suppositionally dismiss supportability of φ and
  (b) every support-alternative for φ is not supposable in *s*
- *s* dismisses a supposition of $\Diamond\varphi$ iff
  (a) there is no support-alternative for φ or
  (b) some support-alternative for φ is not supposable in *s*

Suppositional *might*: persistence

Two essential features of the clauses for $\Diamond\varphi$

- Support and dismissing a supposition contradict each other
- Rejection implies dismissal

Support of *might* is defeasible

- It can be the case that $s \models^+ \Diamond\varphi$ and that it holds for some more informed state $t \subset s$ that $t \not\models^+ \Diamond\varphi$, or even $t \models^- \Diamond\varphi$, but then it will also be the case that $t \models^\circ \Diamond\varphi$.
- Suppositional *might* is support-persistent, modulo suppositional dismissal.

Details of the rejection clauses

- s rejects $\Diamond\varphi$ iff

  (a) s does not suppositionally dismiss supportability of φ and

  (b) every support-alternative for φ is not supposable in s

- Clause (a) restricts clause (b), filtering out cases where not rejection, but only suppositional dismissal is at stake.
- Consider $\Diamond(p \to q)$. Let $s = |\neg p|$.
- The one support-alternative for $p \to q$ is not supposable in s.
- So, s dismisses a supposition of $\Diamond(p \to q)$.
- But s does not reject $\Diamond(p \to q)$, because s also suppositionally dismisses (supportability of) $p \to q$:
- After all, s dismisses a supposition of $p \to q$, no substate of s supports $p \to q$, and s does not reject $p \to q$

Details of the rejection clauses

- s rejects $\Diamond\varphi$ iff
  (a) s does not suppositionally dismiss supportability of φ and
  (b) every support-alternative for φ is not supposable in s

- Consider $\Diamond((p \to q) \vee r)$. Let $s = |\neg p \wedge \neg r|$.
- The two support-alternatives for $(p \to q) \vee r$ are not supposable in s.
- So, s dismisses a supposition of $\Diamond((p \to q) \vee r)$.
- But s does not reject $\Diamond((p \to q) \vee r)$, because s also suppositionally dismisses (supportability of) $(p \to q) \vee r$:
- After all, s dismisses a supposition of $(p \to q) \vee r$, no substate of s supports $(p \to q) \vee r$, and s does not reject $(p \to q) \vee r$.

Suppositional *might* spelled out

$$s \models^{+} \Diamond\varphi \quad \text{iff} \quad \text{ALT}[\varphi]^{+} \neq \emptyset \ \text{ and } \ \forall\alpha \in \text{ALT}[\varphi]^{+} : s \lhd \alpha$$

$$s \models^{-} \Diamond\varphi \quad \text{iff} \quad s \not\models^{\circ}_{\lightning} \varphi \ \text{ and } \ \forall\alpha \in \text{ALT}[\varphi]^{+} : s \ntriangleleft \alpha$$

$$s \models^{\circ} \Diamond\varphi \quad \text{iff} \quad \text{ALT}[\varphi]^{+} = \emptyset \ \text{ or } \ \exists u \in \text{ALT}[\varphi]^{+} : s \ntriangleleft \alpha$$

Reductions

- If φ is support-convex: $s \lhd \alpha \rightsquigarrow \alpha \cap s \models^{+} \varphi$
- If φ is support-dense: $s \lhd \alpha \rightsquigarrow \alpha \cap s \neq \emptyset$
- If φ is support-dense: $s \not\models^{\circ}_{\lightning} \varphi \rightsquigarrow s \neq \emptyset$

Suppositional *might*, reduction for non-inquisitive φ

$$s \models^{+} \Diamond\varphi \quad \text{iff} \quad \text{info}(\varphi) \neq \emptyset \ \text{ and } \ s \lhd \text{info}(\varphi)$$

$$s \models^{-} \Diamond\varphi \quad \text{iff} \quad s \not\models^{\circ}_{\lightning} \varphi \ \text{ and } \ s \ntriangleleft \text{info}(\varphi)$$

$$s \models^{\circ} \Diamond\varphi \quad \text{iff} \quad \text{info}(\varphi) = \emptyset \ \text{ or } \ s \ntriangleleft \text{info}(\varphi)$$

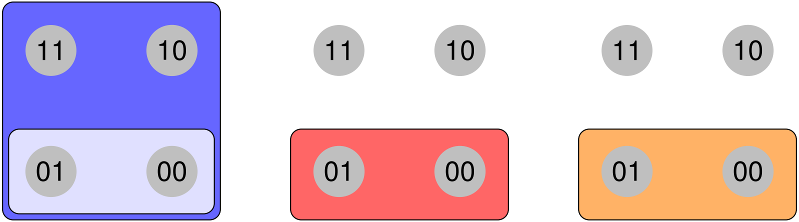
Further reductions

- If φ is support-convex: $s \lhd \text{info}(\varphi) \rightsquigarrow \text{info}(\varphi) \cap s \models^{+} \varphi$
- If φ is support-dense: $s \lhd \text{info}(\varphi) \rightsquigarrow \text{info}(\varphi) \cap s \neq \emptyset$
- If φ is support-dense: $s \not\models^{\circ}_{\lightning} \varphi \rightsquigarrow s \neq \emptyset$

Picture of meaning *might p*

Reduced clauses for $\Diamond p$

- $s \models^{+} \Diamond p$ iff $|p| \cap s \neq \emptyset$
- $s \models^{-} \Diamond p$ iff $|p| \cap s = \emptyset$ and $s \neq \emptyset$
- $s \models^{\circ} \Diamond p$ iff $|p| \cap s = \emptyset$



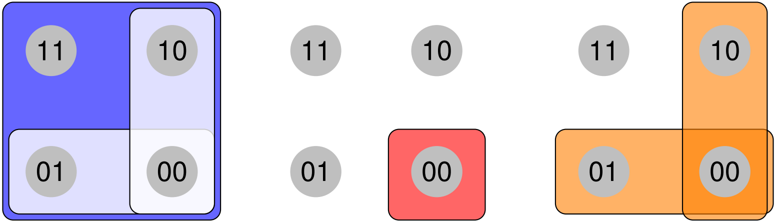
(a) support (b) reject (c) dismissal

$\Diamond p$

Epistemic free choice

Reduced clauses for $\Diamond(p \vee q)$

- $s \models^{+} \Diamond(p \vee q)$ iff $|p| \cap s \neq \emptyset$ and $|q| \cap s \neq \emptyset$
- $s \models^{-} \Diamond(p \vee q)$ iff $|p| \cap s = \emptyset$ and $|q| \cap s = \emptyset$ and $s \neq \emptyset$
- $s \models^{\circ} \Diamond(p \vee q)$ iff $|p| \cap s = \emptyset$ or $|q| \cap s = \emptyset$

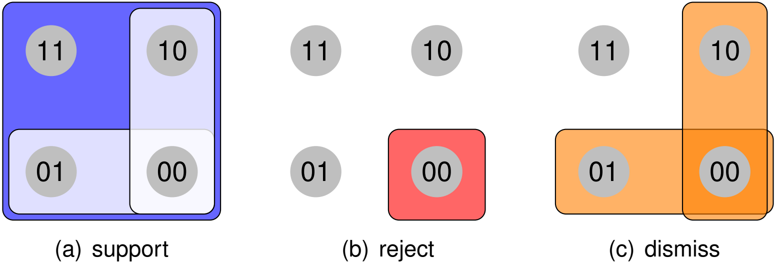


(a) support (b) reject (c) dismiss

$\Diamond(p \vee q)$

Epistemic free choice

- $\Diamond(p \vee q) \models^{+} \Diamond p \wedge \Diamond q$
- $\Diamond(p \vee q) \not\models^{+} \Diamond(p \wedge q)$



$\Diamond(p \vee q)$

2.2. Epistemic *must* as a non-supposability check

Derived suppositional *must*

Must as a non-supposability check

- We standardly define *must* as the dual of *might*: $\Box\varphi := \neg\Diamond\neg\varphi$.
- So, $\Box\varphi$ is supported in *s*, when $\Diamond\neg\varphi$ is rejected in *s*
- $\Diamond\neg\varphi$ is a proposal to check for supposability of $\neg\varphi$ in *s*.
- When the check for supposability of $\neg\varphi$ fails in *s*, $\Diamond\neg\varphi$ is rejected in *s* and $\Box\varphi$ is supported in *s*.
- In InqS, then, $\Box\varphi$ induces a *non*-supposability check of $\neg\varphi$.
- Conversationally, a speaker uttering $\Box\varphi$, invites a responder to suppose that $\neg\varphi$, in the hope that in her state $\neg\varphi$ is (also) not supposable.

Reminder

Suppositionally dismissing rejectability

- $s \models^{\circ}_{\surd} \varphi$ iff $s \models^{\circ} \varphi$ and $\forall t \subseteq s : t \not\models^{-} \varphi$ and $s \not\models^{+} \varphi$.

For non-suppositional φ:

- $s \models^{\circ}_{\surd} \varphi$ iff $s = \emptyset$.

Generally:

- If $s \models^{\circ}_{\surd} \varphi$, then no reject-alternative for φ is supposable in s.

Suppositional *must*: intuitive idea derived from *might*

$\Box\varphi$ is a proposal to check the non-supposability of $\neg\varphi$ in s

- s supports $\Box\varphi$ iff
  (a) s does not suppositionally dismiss rejectability of φ and
  (b) every rejection-alternative for φ is not supposable in s

- s rejects $\Box\varphi$ iff
  (a) there is at least one rejection-alternative for φ and
  (b) every rejection-alternative for φ is supposable in s

- s dismisses a supposition of $\Box\varphi$ iff
  (a) there is no rejection-alternative for φ or
  (b) some rejection-alternative for φ is not supposable in s

Suppositional *must*: support and dismissal

Support implies suppositional dismissal

- s supports $\Box\varphi$ iff
  - (a) s does not suppositionally dismiss rejectability of φ and
  - (b) every rejection-alternative for φ is not supposable in s
- s dismisses a supposition of $\Box\varphi$ iff
  - (a) there is no rejection-alternative for φ or
  - (b) some rejection-alternative for φ is not supposable in s

Suppositional *must*: rejection and dismissal

Rejection and dismissing a supposition contradict each other

- s rejects $\Box\varphi$ iff
  - (a) there is at least one rejection-alternative for φ and
  - (b) every rejection-alternative for φ is supposable in s
- s dismisses a supposition of $\Box\varphi$ iff
  - (a) there is no rejection-alternative for φ or
  - (b) some rejection-alternative for φ is not supposable in s

Suppositional *must*: persistence

Two essential features of the clauses for $\Box\varphi$

- Rejection and dismissing a supposition contradict each other
- Support implies dismissal

Rejection of *must* is defeasible

- It can be the case that $s \models^{-} \Box\varphi$ and that it holds for some more informed $t \subset s$ that $t \not\models^{-} \Box\varphi$, or even $t \models^{+} \Box\varphi$, but then it will also be the case that $t \models^{\circ} \Box\varphi$.
- Suppositional *must* is rejection-persistent, modulo suppositional dismissal.

Details of the support clause

- s supports $\Box\varphi$ iff

  (a) s does not suppositionally dismiss rejectability of φ and

  (b) every rejection-alternative for φ is not supposable in s

- Clause (a) restricts clause (b), filtering out cases where not support, but only suppositional dismissal is at stake.
- Consider $\Box(p \to q)$. Let $s = |\neg p|$.
- The single rejection-alternative for $p \to q$, i.e., $|p \to \neg q|$, is not supposable in s.
- So, s dismisses a supposition of $\Box(p \to q)$.
- But s does not support $\Box(p \to q)$, because s also suppositionally dismisses (rejectability of) $p \to q$.
- After all, s dismisses a supposition of $p \to q$, no substate of s rejects $p \to q$, and s does not support $p \to q$.

Details of the support clause

- s supports $\Box\varphi$ iff
  - (a) s does not suppositionally dismiss rejectability of φ and
  - (b) every rejection-alternative for φ is not supposable in s

- Consider $\Box((p \to q) \wedge r)$. Let $s = |\neg p \wedge r|$.
- The two rejection-alternatives for $(p \to q) \wedge r$, i.e., $|p \to \neg q|$ and $|\neg r|$, are not supposable in s.
- So, s dismisses a supposition of $\Box((p \to q) \wedge r)$.
- But s does not support $\Box((p \to q) \wedge r)$, because s also suppositionally dismisses (rejectability of) $(p \to q) \wedge r$.
- After all, s dismisses a supposition of $(p \to q) \wedge r$, no substate of s rejects $(p \to q) \wedge r$, and s does not support $(p \to q) \wedge r$.

Suppositional epistemic *must* spelled out

$$s \models^{+} \Box\varphi \quad \text{iff} \quad s \not\models^{\circ}_{\surd} \varphi \;\text{ and }\; \forall\alpha \in \textsc{alt}[\varphi]^{-} : s \ntriangleleft \alpha$$

$$s \models^{-} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} \neq \emptyset \;\text{ and }\; \forall\alpha \in \textsc{alt}[\varphi]^{-} : s \triangleleft \alpha$$

$$s \models^{\circ} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} = \emptyset \;\text{ or }\; \exists\alpha \in \textsc{alt}[\varphi]^{-} : s \ntriangleleft \alpha$$

Reductions

- If φ is reject-convex: $s \triangleleft \alpha \;\rightsquigarrow\; \alpha \cap s \models^{-} \varphi$
- If φ is reject-dense: $s \triangleleft \alpha \;\rightsquigarrow\; \alpha \cap s \neq \emptyset$
- If φ is reject-dense: $s \not\models^{\circ}_{\surd} \varphi \rightsquigarrow s \neq \emptyset$

Suppositional *must*: for not reverse-inquisitive φ

$$s \models^+ \Box\varphi \quad \text{iff} \quad \bigcup[\varphi]^- \neq \emptyset \ \text{ and } \ s \lhd \bigcup[\varphi]^-$$

$$s \models^- \Box\varphi \quad \text{iff} \quad s \not\models^\circ_{\surd} \varphi \ \text{ and } \ s \ntriangleleft \bigcup[\varphi^-$$

$$s \models^\circ \Box\varphi \quad \text{iff} \quad \bigcup[\varphi]^- = \emptyset \ \text{ or } \ s \ntriangleleft \bigcup[\varphi]^-$$

Further reductions

- If φ is reject-convex: $s \lhd \bigcup[\varphi]^- \ \rightsquigarrow \ \bigcup[\varphi]^- \cap s \models^- \varphi$
- If φ is reject-dense: $s \lhd \bigcup[\varphi]^- \ \rightsquigarrow \ \bigcup[\varphi]^- \cap s \neq \emptyset$
- If φ is reject-dense: $s \not\models^\circ_{\surd} \varphi \rightsquigarrow s \neq \emptyset$

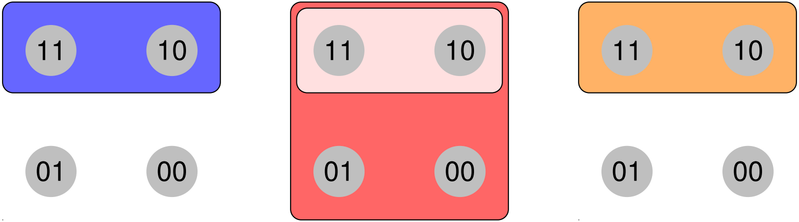
Picture of meaning *must p*

Reduced clauses for $\Box p$

$$s \models^{+} \Box p \quad \text{iff} \quad |\neg p| \cap s = \emptyset \ \text{ and } \ s \neq \emptyset$$

$$s \models^{-} \Box p \quad \text{iff} \quad |\neg p| \cap s \neq \emptyset$$

$$s \models^{\circ} \Box p \quad \text{iff} \quad |\neg p| \cap s = \emptyset$$



(a) support (b) reject (c) dismiss

$\Box p$

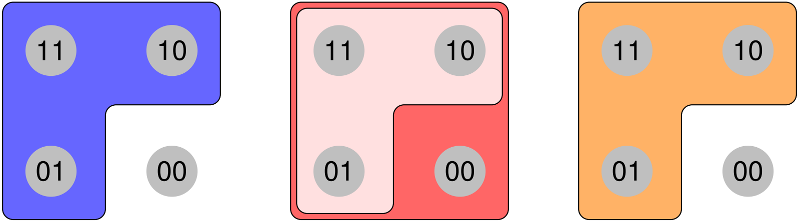
Picture of meaning *must* $(p \vee q)$

Reduced clauses for $\Box(p \vee q)$

$$s \models^{+} \Box(p \vee q) \quad \text{iff} \quad |\neg p \wedge \neg q| \cap s = \emptyset \quad \text{and} \quad s \neq \emptyset$$

$$s \models^{-} \Box(p \vee q) \quad \text{iff} \quad |\neg p \wedge \neg q| \cap s \neq \emptyset$$

$$s \models^{\circ} \Box(p \vee q) \quad \text{iff} \quad |\neg p \wedge \neg q| \cap s = \emptyset$$



(a) support (b) reject (c) dismiss

$\Box(p \vee q)$

2.3. Non-inquisitive closure by *might* and *must*

Suppositional *must* and non-inquisitive closure

- The reject-informative content of $\Box\varphi$ is nil:

$$\bigcup[\Box\varphi]^{-} = \omega$$

- The support-informative content of $\Box\varphi$ equals that of φ:

$$\bigcup[\Box\varphi]^{+} = \bigcup[\varphi]^{+}$$

- But it does not hold generally that $[\Box\varphi]^{+} = [\varphi]^{+}$.

$$\textsc{alt}[p \vee q]^{+} = \{|p|, |q|\} \quad \neq \quad \textsc{alt}[\Box(p \vee q)]^{+} = \{|p| \cup |q|\}$$

- $p \vee q$ is support-inquisitive, but $\Box(p \vee q)$ is not.
- $\Box(p \vee \neg p)$ is supported in every state, support of $p \vee \neg p$ requires support of p or support of $\neg p$.

Suppositional *might* and non-inquisitive closure

- The support-informative content of $\Diamond\varphi$ is nil:

$$\bigcup[\Diamond\varphi]^+ = \omega$$

- The reject-informative content of $\Diamond\varphi$ equals that of φ:

$$\bigcup[\Diamond\varphi]^- = \bigcup[\varphi]^-$$

- But it does not hold generally that $[\Diamond\varphi]^- = [\varphi]^-$.

$$\textsc{alt}[p \wedge q]^- = \{|\neg p|, |\neg q|\} \quad \neq \quad \textsc{alt}[\Diamond(p \wedge q)]^- = \{|\neg p| \cup |\neg q|\}$$

- $p \wedge q$ is reject-inquisitive, but $\Diamond(p \wedge q)$ is not.
- $\Diamond(p \wedge \neg p)$ is rejected in every state, rejection of $p \wedge \neg p$ requires rejection of p or rejection of $\neg p$.

2.4. Modal and non-modal implications

Modal and non-modal implications

Rejecting implication

- In InqS, not just $p \wedge \neg q$, but also $p \to \neg q$ rejects $p \to q$.
- Some may feel this is still asking too much, and that $p \to \Diamond\neg q$ or $\Diamond(p \wedge \neg q)$ should already suffice to reject $p \to q$.
- But neither of these responses is support-informative, they are already supported by the ignorant state ω.
- But sheer ignorance about p and q should not suffice to reject the proposal to update the CG with the information that $p \to q$.
- Responding with $p \to \Diamond\neg q$ or $\Diamond(p \wedge \neg q)$ to $p \to q$, signals unwillingness and not unability to accept the proposal.

Modal and non-modal implications

Rejecting implication continued

- Both $p \to \Diamond\neg q$ and $\Diamond(p \wedge \neg q)$ do suffice to reject $p \to \Box q$.
- By proposing $p \to \Box q$ instead of $p \to q$, one signals that ignorance about p and q suffices to reject the proposal.
- One only intends an update of the CG with $p \to q$, in case the other participants also already support that $p \to q$ or $p \to \Box q$.

Implication in natural language

- InqS as such is neutral as to whether NL-conditionals should generally be analyzed as modal or non-modal implications.
- What matters to us here are the inquisitive and suppositional features of the semantics.

Final remark

- One obvious question to ask is whether the semantics of epistemic modalities presented here can be extended to, e.g., deontic modalities.
- The latter have been studied by Martin Aher in his PhD-thesis within the framework of radical inquisitive semantics.
- He proposes a “modified Andersonian analysis” of deontic modalities, in which they are intimately linked with implication.
- In a joint talk we have ‘lifted’ this analysis to InqS, accounting simultaneously for both types of modalities, showing the structural similarities between the semantics of both types of modalities and the semantics of implication in InqS.
- The combined forces of both types of modalities shed new light on several of the “deontic puzzles” that have been discussed in the literature.

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